

# Causal Strategic Learning with Competitive Selection

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## (0). TL; DR

Prior work studied predictions (incl. classification) but did not consider the impact of selection.

We show that:

- The optimal selection rule is a trade-off between picking the most capable agents and maximally incentivising everyone,
- Agent improvement requires not only causal inference but also a benevolent regulator to enforce it (see (3). and (6).).

## (2). Core assumptions

- Linear & additive structures.
- Agents differ in their base covariates, but improve by the same amount.

## (3). Minimising the trade-off

Additive structures result in:

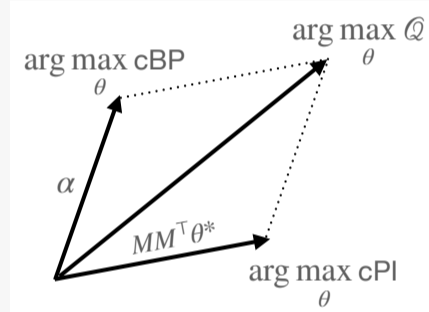
$$Q(\theta) = \text{cBP}(\theta) + \text{cPI}(\theta)$$

Further linearity leads to:

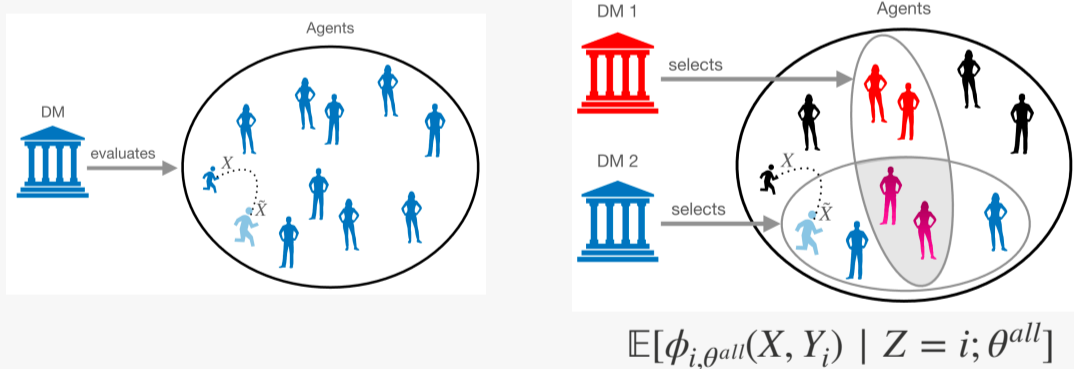
$$\arg \max_{\theta} Q(\theta) = \frac{\alpha + MM^T \theta^*}{\|\alpha + MM^T \theta^*\|}$$

To align the two objectives:

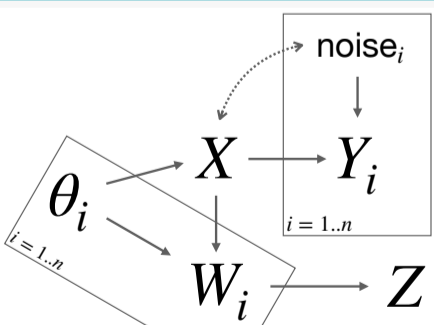
$$\alpha = (k-1)MM^T \theta^*$$



## (5). Prior work vs Ours (cont'd)



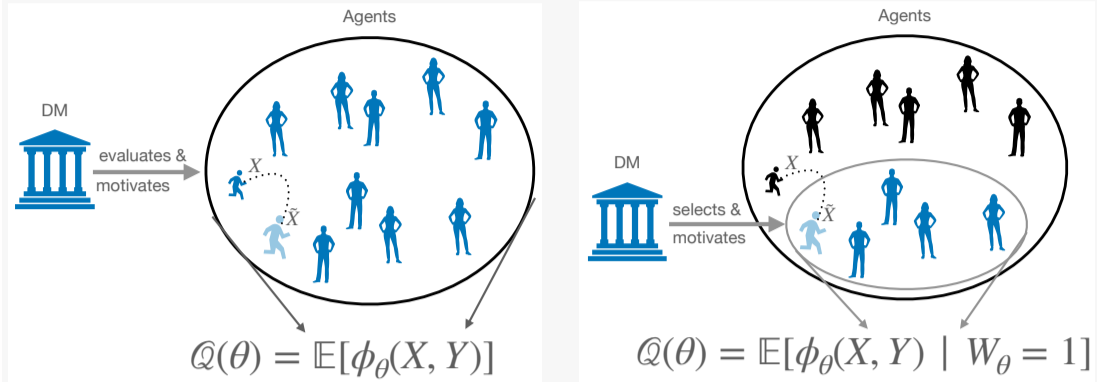
## (6). Learning under competitive selection



A cooperative protocol for all DMs to obtain partitions of data without correlated noise:

$$\exists \{t, t'\}, \forall i : \theta_{it} = k_i \theta_{it'}$$

## (1). Prior work vs. Ours...



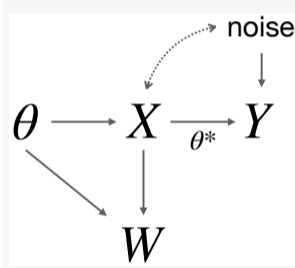
for some objective function  $\phi_{\theta}(X, Y)$  encompassing BP (base performance) and PI (performance improvement).

With additive structures:

$$\begin{aligned} \mathbb{E}[\phi_{\theta}(X, Y)] &= \mathbb{E}[\text{BP}] + \mathbb{E}[\text{PI}_{\theta}] \\ &= \text{const} + \mathbb{E}[\text{PI}_{\theta}] \end{aligned}$$

$$\begin{aligned} \mathbb{E}[\phi_{\theta}(X, Y) | W_{\theta} = 1] &= \mathbb{E}[\text{BP} | W_{\theta} = 1] + \mathbb{E}[\text{PI}_{\theta} | W_{\theta} = 1] \\ &= \text{cBP}(\theta) + \text{cPI}(\theta) \end{aligned}$$

## (4). Learning $\theta^*$ under selection bias



$\exists \{\theta_{\dagger}, \theta_{\diamond}\} :$

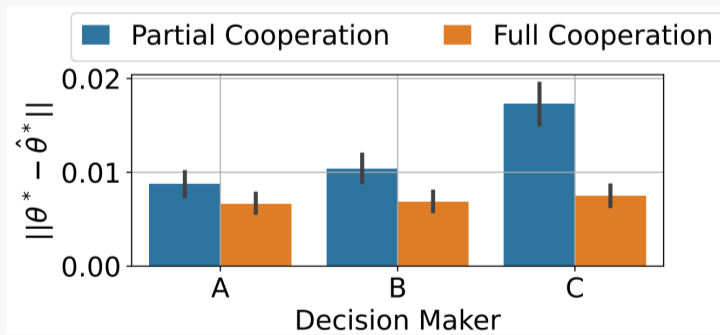
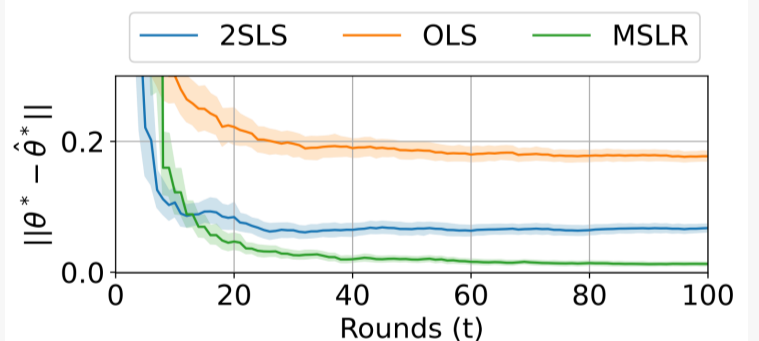
$$P(\text{noise} | W = 1; \theta_{\dagger}) = P(\text{noise} | W = 1; \theta_{\diamond})$$

$$\Delta \theta \rightarrow \Delta \mathbb{E}[X | W = 1] \xrightarrow{\theta^*} \Delta \mathbb{E}[Y | W = 1]$$

Certain partitioning schemes can remove the correlated noise.

## (7). Experiments

Our MSLR has unbiased estimation of  $\theta^*$ .



More cooperation, less estimation error.



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